

Motivation
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Identities of operators
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Metaplectic Tokuyama
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Metaplectic Kac-Moody
oooo

Correction factor
ooooo

The Chinta-Gunnells action and sums over highest weight crystals

Anna Puskás

University of Massachusetts, Amherst

SageDays@ICERM: Combinatorics and Representation Theory
July 23, 2018

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4 Metaplectic Kac-Moody

5 Correction factor

Notation

- Φ root system, $\alpha_1, \dots, \alpha_r$ simple roots
 - $\sigma_1, \dots, \sigma_r$ simple reflections, W Weyl group
 - Λ weight lattice, $\mathbb{C}_v(\Lambda)$
 - F, q, G, K, U
 - n positive integer
-
- Demazure, Demazure-Lusztig operators $\mathcal{D}_w, \mathcal{T}_w$ ($w \in W$)
 - Sums in $\mathbb{C}_v(\Lambda)$
 - Highest weight crystals $\mathcal{B}_{\lambda+\rho}$
 - Symmetrizers $\sum_w \mathcal{T}_w$

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Whittaker functions

$\mathcal{W} : G \rightarrow \mathbb{C}$ is determined by certain invariance properties, and a character of F^+ .

The Shintani Casselman-Shalika formula computes Whittaker functions.

$$\mathcal{W}(\pi^\lambda) = q^{-\langle \rho, \lambda \rangle} \cdot \Delta_{q^{-1}} \cdot \chi_\lambda(x)$$

Generalizations to the metaplectic /affine case?

- Metaplectic: $1 \rightarrow \mu_n \rightarrow \tilde{G} \rightarrow G \rightarrow 1$
- Affine and beyond: Φ, W infinite.

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Sum over Weyl group

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Sum over Weyl group

$$\frac{\Delta_v}{\Delta_1} \sum_{w \in W} \text{sgn}(w) w(e^{\lambda+\rho})$$

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Sum over crystal

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Sum over Weyl group

$$\frac{\Delta_v}{\Delta_1} \sum_{w \in W} \text{sgn}(w) w(e^{\lambda + \rho})$$

Sum over crystal

$$\sum_{b \in \mathcal{B}_{\lambda + \rho}} G(b) \cdot e^{\text{wt}(b)}$$

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Hecke symmetrizer

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Hecke symmetrizer

$$\left(\sum_{w \in W} \mathcal{T}_w \right) e^\lambda$$

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$$\left(\sum_{w \in W} T_w \right) e^\lambda$$

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Identities of operators

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$$\left(\sum_{w \in W} T_w \right) e^\lambda$$

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Sum over crystal

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Identities of operators

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Sum over crystal

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Identities of operators

Induction on rank

Hecke symmetrizer

$$\left(\sum_{w \in W} T_w \right) e^\lambda$$

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Tokuyama's Theorem

$$\prod_{1 \leq i < j < r+1} (x_j - v \cdot x_i) \cdot s_\lambda(\mathbf{x}) = \sum_{b \in \mathcal{B}_{\lambda+\rho}} G(b) \cdot \mathbf{x}^{wt(b)}.$$

Schur function $s_\lambda(\mathbf{x})$

Crystal $\mathcal{B}_{\lambda+\rho}$

Young diagram of λ : 

Weighted Young diagram of λ : 

Weighted Young diagram of $\lambda + \rho$: 



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Sum over the group S_{r+1} :

$$\frac{\Delta_v}{\Delta} \cdot \sum_{w \in S_{r+1}} \text{sgn}(w) \cdot w(\mathbf{x}^{\lambda+\rho})$$

Crystal $\mathcal{B}_{\lambda+\rho}$

Position of b in $\mathcal{B}_{\lambda+\rho}$ gives $G(b)$



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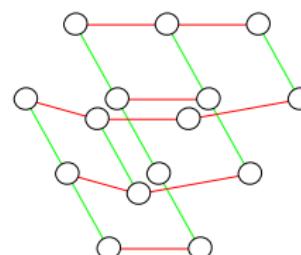
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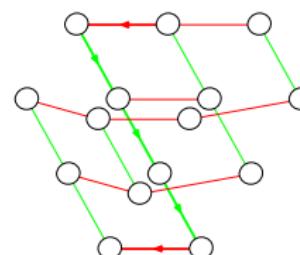
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Metaplectic Casselman-Shalika

$$\frac{\widetilde{\Delta_v}}{\widetilde{\Delta_1}} \cdot \sum_{w \in S_{r+1}} \operatorname{sgn}(w) \cdot w(x^{\lambda+\rho})$$

The action of W on $\mathbb{C}(\Lambda)$ can be modified to depend on n .
(Chinta-Gunnells, Chinta-Offen, McNamara)

$$\sum_{b \in \mathcal{B}_{\lambda+\rho}} G(b) \cdot x^{\operatorname{wt}(b)}$$

The definition of $G(b)$ can be modified to involve Gauss-sums
(modulo n).
(Brubaker, Bump, Friedberg, McNamara, Zhang)

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Sum over Weyl group

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Sum over crystal

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Iwahori-Whittaker

$$\mathcal{W}_{w, \lambda^\vee} \approx \mathcal{T}_w x^{\lambda^\vee}$$

Iwahori-Whittaker functions and the \mathcal{T}_w

Brubaker-Bump-Licata:

- express the value of a Whittaker functional on Iwahori-fixed vectors of a principal series representation as $\mathcal{T}_w x^{\lambda^\vee}$
- relate identities of $\mathcal{W}_{w,\lambda^\vee} \approx \mathcal{T}_w x^{\lambda^\vee}$ to combinatorics of Bott-Samelson resolutions, non-symmetric Macdonald polynomials
- exploit uniqueness of the Whittaker functional

Patnaik:

- gives a proof of $\mathcal{W}_{w,\lambda^\vee} \approx \mathcal{T}_w x^{\lambda^\vee}$ without exploiting uniqueness
- the method generalizes to the affine Kac-Moody setting

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- relate identities of $\mathcal{W}_{w,\lambda^\vee} \approx \mathcal{T}_w x^{\lambda^\vee}$ to combinatorics of Bott-Samelson resolutions, non-symmetric Macdonald polynomials
- exploit uniqueness of the Whittaker functional

Patnaik:

- gives a proof of $\mathcal{W}_{w,\lambda^\vee} \approx \mathcal{T}_w x^{\lambda^\vee}$ without exploiting uniqueness
- the method generalizes to the affine Kac-Moody setting

Iwahori-Whittaker functions and the \mathcal{T}_w

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Motivation
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Identities of operators
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Metaplectic Tokuyama
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Metaplectic Kac-Moody
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Correction factor
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Metaplectic Demazure operators

Demazure operators: σ_i simple reflection, $f \in \mathbb{C}(\Lambda)$:

$$\mathcal{D}_{\sigma_i}(f) = \frac{f - \mathbf{x}^{-n(\alpha^\vee_i)\alpha^\vee_i}\sigma_i(f)}{1 - \mathbf{x}^{-n(\alpha^\vee_i)\alpha^\vee_i}}$$

Demazure-Lusztig operators:

$$\mathcal{T}_{\sigma_i}(f) = (1 - v \cdot \mathbf{x}^{-n(\alpha^\vee_i)\alpha^\vee_i}) \cdot \mathcal{D}_{\sigma_i}(f) - f$$

$n(\alpha^\vee) = \frac{n}{\gcd(n, ||\alpha^\vee||^2)}$ and $\sigma_i(f)$ is the Chinta-Gunnells action

$\mathcal{D}_{\sigma_i}, \mathcal{T}_{\sigma_i}$ satisfy Braid-relations $\longrightarrow \mathcal{D}_w, \mathcal{T}_w$ for every $w \in W$

Motivation
ooooooo

Identities of operators
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Metaplectic Tokuyama
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Identities of operators
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Metaplectic Tokuyama
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Metaplectic Kac-Moody
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Correction factor
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Identities for the long word

Theorem (Chinta, Gunnells, P.)

$$\mathcal{D}_{w_0} = \frac{1}{\tilde{\Delta}} \cdot \sum_{w \in W} \text{sgn}(w) \cdot \prod_{\alpha \in \Phi(w^{-1})} e^{\text{n}(\alpha)\alpha} \cdot w.$$

$$\widetilde{\Delta}_v \cdot \mathcal{D}_{w_0} = \sum_{w \in W} \mathcal{T}_w.$$

Motivation
ooooooo

Identities of operators
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Metaplectic Tokuyama
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Metaplectic Kac-Moody
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Motivation
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Identities of operators
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Metaplectic Tokuyama
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Motivation
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Identities of operators
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Metaplectic Tokuyama
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Motivation
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Identities of operators
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Metaplectic Tokuyama
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Metaplectic Kac-Moody
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Correction factor
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Motivation
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Identities of operators
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Metaplectic Tokuyama
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Metaplectic Kac-Moody
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Correction factor
ooooo

Sum over Weyl group

$$\frac{\widetilde{\Delta_v}}{\widetilde{\Delta_1}} \sum_{w \in W} \text{sgn}(w) w \left(e^{\lambda + \rho} \right)$$

Motivation
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Identities of operators
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Metaplectic Tokuyama
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Metaplectic Kac-Moody
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Correction factor
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Sum over Weyl group

$$\frac{\widetilde{\Delta_v}}{\widetilde{\Delta_1}} \sum_{w \in W} \text{sgn}(w) w \left(e^{\lambda + \rho} \right)$$

Hecke symmetrizer

$$\sum_{w \in W} T_w$$

Motivation
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Identities of operators
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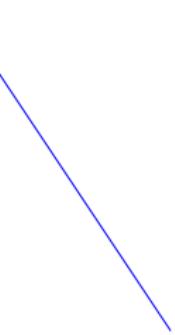
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Metaplectic Kac-Moody
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Correction factor
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Motivation
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Identities of operators
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Metaplectic Tokuyama
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Metaplectic Kac-Moody
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Correction factor
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Motivation
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Identities of operators
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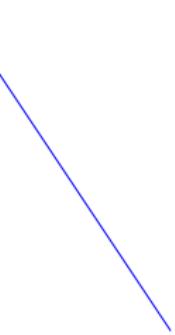
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Correction factor
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Sum over crystal

$$\sum_{b \in \mathcal{B}_{\lambda + \rho}} G(b) \cdot e^{\text{wt}(b)}$$

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Hecke symmetrizer

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Motivation
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Identities of operators
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Metaplectic Tokuyama
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Metaplectic Kac-Moody
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Correction factor
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Theorem (P.)

$$\left(\sum_{u \leq w} T_u \right) \cdot x^{\lambda^\vee} = x^{-\rho} \sum_{b \in \mathcal{B}_{\lambda+\rho}^{(w)}} G(b) x^{\text{wt}(b)}$$

Sum over the Weyl group, bounded in the Bruhat order



Motivation
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Identities of operators
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Metaplectic Tokuyama
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Metaplectic Kac-Moody
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Correction factor
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Theorem (P.)

$$\left(\sum_{u \leq w} T_u \right) \cdot \mathbf{x}^{\lambda^\vee} = \mathbf{x}^{-\rho} \sum_{b \in \mathcal{B}_{\lambda+\rho}^{(w)}} G(b) \mathbf{x}^{\text{wt}(b)}$$

Sum over the Weyl group, bounded in the Bruhat order

Demazure subcrystal



Motivation
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Identities of operators
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Metaplectic Tokuyama
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Metaplectic Kac-Moody
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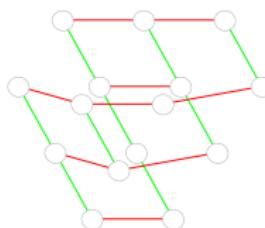
Correction factor
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$\mathcal{B}_{\lambda+\rho}^{(w)}$ Demazure subcrystal



Motivation
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Identities of operators
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Metaplectic Tokuyama
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Metaplectic Kac-Moody
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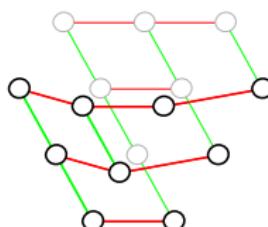
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Motivation
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Identities of operators
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Metaplectic Tokuyama
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Metaplectic Kac-Moody
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Correction factor
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Motivation
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Identities of operators
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Metaplectic Tokuyama
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Metaplectic Kac-Moody
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Correction factor
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Sum over Weyl group

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Sum over crystal

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Hecke symmetrizer

$$\sum_{w \in W} \mathcal{T}_w$$

Motivation
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Identities of operators
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Metaplectic Tokuyama
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Correction factor
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Induction by rank

Hecke symmetrizer

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Motivation
ooooooo

Identities of operators
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Metaplectic Tokuyama
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Metaplectic Kac-Moody
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Correction factor
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Induction by rank



Motivation

Identities of operators

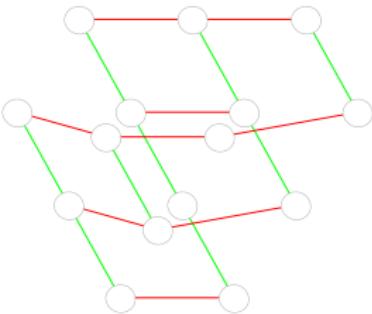
Metaplectic Tokuyama ○○●

Metaplectic Kac-Moody ooooo

Correction factor
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Induction by rank

$$\sum_{w \in W^{(r)}} \mathcal{T}_w = \sum_{w \in W^{(r-1)}} \mathcal{T}_w \cdot (1 + \mathcal{T}_r + \mathcal{T}_r \mathcal{T}_{r-1} + \cdots + \mathcal{T}_r \mathcal{T}_{r-1} \cdots \mathcal{T}_1)$$



Motivation
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Identities of operators
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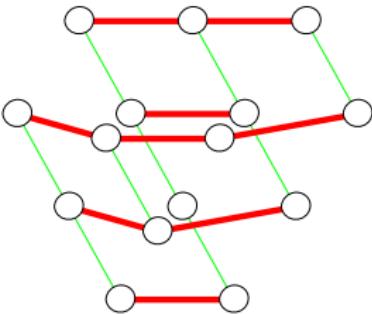
Metaplectic Tokuyama
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Joint work in progress with Paul E. Gunnells:
this technique extends to Cartan type D.

Motivation
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Identities of operators
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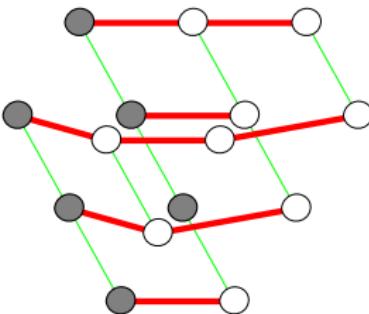
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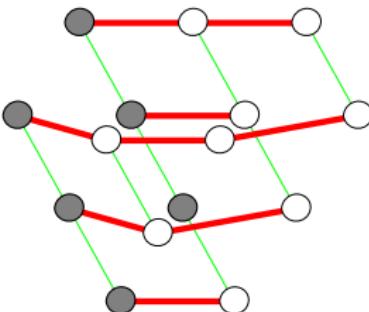
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Correction factor
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Demazure-Lusztig operator \mathcal{T}_w

Questions:

- interpretation as metaplectic Iwahori-Whittaker function
- infinite dimensional case: metaplectic Kac-Moody Whittaker functions
- identities and relationship to Weyl-Kac character

$$\left(\sum_{w \in W} \mathcal{T}_w \right) e^\lambda \sim m \Delta_\nu \chi_\lambda$$

Joint work with Manish Patnaik

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Joint work with Manish Patnaik

Motivation
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Identities of operators
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Metaplectic Tokuyama
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Metaplectic Kac-Moody
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Correction factor
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Metaplectic Iwahori-Whittaker functions $\widetilde{\mathcal{W}}_{u,\lambda^\vee}$

$$\widetilde{\mathcal{W}}(\pi^{\lambda^\vee}) = q^{-2\langle \rho, \lambda^\vee \rangle} \cdot \sum_{u \in W} \widetilde{\mathcal{W}}_{u,\lambda^\vee}$$

Theorem (Patnaik, P.)

Let $w, w' \in W$ and $w = \sigma_i w'$ with $\ell(w) = \ell(w') + 1$:

$$\mathcal{T}_{\sigma_i}(\widetilde{\mathcal{W}}_{w',\lambda^\vee}) = \widetilde{\mathcal{W}}_{w,\lambda^\vee}$$

Corollary: $\widetilde{\mathcal{W}}_{w,\lambda^\vee} = q^{\langle \rho, \lambda^\vee \rangle} \cdot \mathcal{T}_w(e^{\lambda^\vee}).$

(New proof of the metaplectic Casselman-Shalika formula.)

Motivation
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Identities of operators
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Metaplectic Tokuyama
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Motivation
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Identities of operators
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Metaplectic Tokuyama
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Metaplectic Kac-Moody
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Correction factor
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Metaplectic Iwahori-Whittaker functions $\widetilde{\mathcal{W}}_{u,\lambda^\vee}$

$$\widetilde{\mathcal{W}}(\pi^{\lambda^\vee}) = q^{-2\langle \rho, \lambda^\vee \rangle} \cdot \sum_{u \in W} \widetilde{\mathcal{W}}_{u,\lambda^\vee}$$

Theorem (Patnaik, P.)

Let $w, w' \in W$ and $w = \sigma_i w'$ with $\ell(w) = \ell(w') + 1$:

$$\mathcal{T}_{\sigma_i}(\widetilde{\mathcal{W}}_{w',\lambda^\vee}) = \widetilde{\mathcal{W}}_{w,\lambda^\vee}$$

Corollary: $\widetilde{\mathcal{W}}_{w,\lambda^\vee} = q^{\langle \rho, \lambda^\vee \rangle} \cdot \mathcal{T}_w(e^{\lambda^\vee}).$

(New proof of the metaplectic Casselman-Shalika formula.)

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Nonmetaplectic, affine result:

Theorem (Patnaik)

$$\mathcal{W}(\pi^{\lambda^\vee}) = q^{\langle \rho, \lambda^\vee \rangle} \cdot \sum_{u \in W} \mathcal{T}_u(e^{\lambda^\vee}) = \mathfrak{m} \cdot q^{\langle \rho, \lambda^\vee \rangle} \cdot \chi_{\lambda^\vee}$$

Metaplectic context:

- What is the metaplectic cover of a Kac-Moody group?
- Issues with the convergence of

$$\sum_{u \in W} \mathcal{T}_u(e^{\lambda^\vee})$$

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Well-definedness and Convergence

For any $w \in W$ we may expand

$$\mathcal{T}_w = \sum_{u \leq w} A_u(w)[u]$$

Summing this over W :

$$\sum_{w \in W} \mathcal{T}_w = \sum_{w \in W} \sum_{u \leq w} A_u(w)[u]$$

For a fixed $u \in W$, why is $\sum_{u \leq w} A_u(w)$ well-defined?

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Results - Patnaik, P.

- Given $F, (\cdot, \cdot) : F^* \times F^* \rightarrow A$, $G, Q : \Lambda^\vee \rightarrow \mathbb{Z}$, B . There exists

$$1 \rightarrow A \rightarrow E \rightarrow G \rightarrow 1$$

such that restricted to the torus H

$(\lambda^\vee, \mu^\vee \in \Lambda^\vee, s, t \in F^*, s^{\lambda^\vee}, t^{\mu^\vee} \in H)$:

$$[s^{\lambda^\vee}, t^{\mu^\vee}] = (s, t)^{B(\lambda^\vee, \mu^\vee)}.$$

$$\widehat{W}(\pi^{\lambda^\vee}) = m_{\Phi_n^\vee} \Delta_{\Phi_n^\vee} \sum_{w \in W} (-1)^{\ell(w)} \left(\prod_{\tilde{\alpha}^\vee \in \Phi_n^\vee(w)} e^{-\tilde{\alpha}^\vee} \right) w * e^{\lambda^\vee},$$

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$$\sum_{w \in W} \mathcal{T}_w = \mathfrak{m}_\Phi \Delta_\Phi \sum_{w \in W} (-1)^{\ell(w)} \left(\prod_{\alpha \in \Phi(w)} e^{-\alpha} \right) w$$

$$\mathfrak{m}_\Phi \cdot \sum_{w \in W} w \left(\frac{\Delta_v}{\Delta} \right) = \sum_{w \in W} v^{\ell(w)}$$

- Φ finite type: $m = 1$
- Φ affine type:
 - supported on Φ_{imag}
 - known by Cherednik's proof of Macdonald constant term conjecture
- Beyond affine type: polynomiality by Viswanath

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$$\sum_{w \in W} \mathcal{T}_w = \textcolor{blue}{m_\Phi} \Delta_\Phi \sum_{w \in W} (-1)^{\ell(w)} \left(\prod_{\alpha \in \Phi(w)} e^{-\alpha} \right) w$$

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Joint work with Dinakar Muthiah and Ian Whitehead:

$$\textcolor{blue}{m} \sum_{w \in W} w \left(\prod_{\alpha \in \Phi_{\text{real}}^+} \frac{1 - ve^\alpha}{1 - e^\alpha} \right) = \sum_{w \in W} v^{\ell(w)}$$

$$\textcolor{brown}{m} = \prod_{\lambda \in Q_{\text{imag}}^+} \prod_{k \geq 0} (1 - v^n e^\lambda)^{-m(\lambda, k)}$$

$$m_\lambda(v) = \sum_{k \geq 0} m(\lambda, k) v^k$$

The $m_\lambda(0)$ are the root multiplicities.

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Results - Muthiah, P., Whitehead

- Generalized Peterson algorithm to compute $m(\lambda, k)$
(for $\lambda \in Q_{\text{imag}}^+$, $k \geq 0$ from Φ_{real})
- Generalized Berman-Moody formula

$$m_\lambda(v) = \sum_{\kappa|\lambda} \mu(\lambda/\kappa) \left(\frac{\lambda}{\kappa}\right)^{-1} \sum_{\underline{\kappa} \in \text{Par}(\kappa)} (-1)^{|\underline{\kappa}|} \frac{B(\underline{\kappa})}{|\underline{\kappa}|} \prod_{i=1}^{|\underline{\kappa}|} P_{\kappa_i}(v^{\lambda/\kappa})$$

- For all $\lambda \in Q_{\text{imag}}^+$, $m_\lambda(v)$ is nonzero if and only if λ is a root.
- For imaginary roots λ , the polynomial $m_\lambda(v)$ is divisible by $(1-v)^2$.

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- For all $\lambda \in Q_{\text{imag}}^+$, $m_\lambda(v)$ is nonzero if and only if λ is a root.
- For imaginary roots λ , the polynomial $m_\lambda(v)$ is divisible by $(1 - v)^2$.

Results - Muthiah, P., Whitehead

- Generalized Peterson algorithm to compute $m(\lambda, k)$
(for $\lambda \in Q_{\text{imag}}^+$, $k \geq 0$ from Φ_{real})
- Generalized Berman-Moody formula

$$m_\lambda(v) = \sum_{\kappa|\lambda} \mu(\lambda/\kappa) \left(\frac{\lambda}{\kappa}\right)^{-1} \sum_{\underline{\kappa} \in \text{Par}(\kappa)} (-1)^{|\underline{\kappa}|} \frac{B(\underline{\kappa})}{|\underline{\kappa}|} \prod_{i=1}^{|\underline{\kappa}|} P_{\kappa_i}(v^{\lambda/\kappa})$$

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Motivation
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Identities of operators
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Metaplectic Tokuyama
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Metaplectic Kac-Moody
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Correction factor
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Further questions

On the correction factor m :

- Interpretation of all coefficients of $\frac{m_\lambda(v)}{(1-v)^2}$ in terms of the Kac-Moody Lie algebra.
- Conjecture: in rank two hyperbolic type, $\frac{m_\lambda(v)}{(1-v)^2}$ have alternating sign coefficients.
- Give upper bounds for the degree and coefficients of $\frac{m_\lambda(v)}{(1-v)^2}$

On the constructions of metaplectic Iwahori-Whittaker functions:

- Understand combinatorial descriptions in every finite Cartan-type
- Extend combinatorial constructions to the affine or general Kac-Moody setting
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Thank you!